Section 2.5

The Fundamental Theorem of Algebra

If $\mathrm{f(x)}$ is a polynomial of degree n, where $\mathrm{n} > 0$, then f has at least one zero in the complex number system.

Linear Factorization Theorem

If $\mathrm{f(x)}$ is a polynomial of degree n, where $\mathrm{n} > 0$, then f has precisely n linear factors

$$f(x) = a_n(x - c_1)(x - c_2) \cdots (x - c_n)$$

where $c_1, c_2, \ldots, c_n$ are complex numbers.

The Rational Zero Test

If the polynomial $f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_2x^2 + a_1x + a_0$ has integer coefficients, every rational zero of f has the form

$$\text{Rational zero} = \frac{p}{q}$$

where p and q have no common factors other than 1, and

$$p = \text{a factor of the constant term } a_0$$
$$q = \text{a factor of the leading coefficient } a_n$$

Complex Zeros Occur in Conjugate Pairs

Let $f(x)$ be a polynomial function that has real coefficients. If $a + bi$, where $b \neq 0$, is a zero of the function, the conjugate $a - bi$ is also a zero of the function.

Problem 1. Find all rational zeros of the function, and write the polynomial as a product of linear factors.

a) $f(x) = x^3 - 10x^2 + 31x - 30$

b) $f(x) = 2x^3 - 9x^2 + 12x - 4$

c) $f(x) = 2x^4 - 9x^3 + 7x^2 + 9x - 9$

Problem 2. Use the given zero to find all the zeros of the function, and write the polynomial as a product of linear factors.

a) $f(x) = x^3 - x^2 + 4x - 4,\ x = 2i$

b) $x^3 - 4x^2 + x + 26,\ x = 3 + 2i$

c) $f(x) = x^4 + 2x^3 - 4x^2 - 26x - 21,\ x = -2 + \sqrt{3}i$

Homework: Read section 2.5, do #17, 21, 27, 29, 37, 43, 47, 53, 59, 71, 77, 85